The Standard Deviants Presents

The Supersonic World of
Differential Equations

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IV. Variations of Parameters

Derivative Form: $y + y = 0$

Differential Form: $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Differential Operator Form: $D^2y + Dy = 0$

Separable Differential Equation: $\frac{dy}{dx} = f(x)g(y)$

Exact Differential Equation: $M(x,y)dx + N(x,y)dy = 0,$ where $\frac{M}{y} = \frac{N}{x}$

Linear First Order Differential Equation: $\frac{dy}{dx} + P(x)y = Q(x)$

- Formula for Integrating Factor: $I(x) = e^{P(x)dx}$

Homogeneity: $ay + by + cy = g(x)$

If $g(x) = 0$ homogeneous
If $g(x)$ π 0 non-homogeneous

General Solution Form for Second Order Differential Equation:

$y = c_1 \; solution_1 + c_2 \; solution_2$

- Solution Form:
  $y = e^{mx}$

- Possible Outcomes:
  - 2 Distinct Real Roots
  - Repeated Roots
  - Complex Roots $(m = a \quad bi)$
    Use $y = e^{ax} \cos bx$, and $y = e^{ax} \sin bx$

Non-homogeneous Differential Equations with Constant Coefficients:

- Final Solution Form: $y = y_c + y_p$
  y_c = complementary solution
  y_p = particular solution

Variations of Parameters Formulas:

$y_p = Ay_1 + By_2$

$A = -\frac{g(x)\, y_2}{w}$ and $B = \frac{g(x)\, y_1}{w}$, where $w = y_1 y_2 - y_2 y_1$

$A = -\frac{g(x)y_2}{w}$ and $B = \frac{g(x)y_1}{w}$

Chart to Choose y_p

form of g(x)	*form of y_p*
constant	A
$3x + 1$	$Ax + B$
$3x^2$	$Ax^2 + Bx + C$
$3x^3 + x^2 + 1$	$Ax^3 + Bx^2 + Cx + D$
$\sin x$	$A\cos x + B\sin x$
$\cos x$	$A\cos x + B\sin x$
e^{mx}	Ae^{mx}
xe^{mx}	$(Ax + B)e^{mx}$
$(x^2 + 1)e^{mx}$	$(Ax^2 + Bx + C)e^{mx}$
$e^{mx}\sin x$	$Ae^{mx}\cos x + Be^{mx}\sin x$
$x^2\sin x$	$(Ax^2 + Bx + C)\cos x + (Dx^2 + Ex + F)\sin x$
$xe^{mx}\cos x$	$(Ax + B)e^{mx}\cos x + (Cx + D)e^{mx}\sin x$

Useful Rules, Formulas, and Identities:

$e^{a+b} = (e^a)(e^b)$

$x = \frac{-b \quad b^2-4ac}{2a}$ (Quadratic Formula)

$\frac{1}{y}\,dy = \Omega 1n\ y\Omega + C$

$udv = uv - \quad vdu$ (Integration by Parts)

$\cos xdx = \sin x + C$

$\sin xdx = -\cos x + C$

$\sec^2 xdx = \tan x + C$

$\sec xdx = 1n\Omega\sec x + \tan x\Omega + C$

$\sec x \tan xdx = \sec x + C$

$\frac{1}{y^2+1}\,dy = \arctan y + C$ or $\tan^{-1} y + C$

$\frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C$

$\frac{1}{a^2-x^2}\,dx = \arcsin\frac{x}{a} + C$

$\cos^2 x + \sin^2 x = 1$

$\sec x = \frac{1}{\cos x}$

$\frac{\sin x}{\cos x} = \tan x$